The Form of the Cosmic Gamma-Ray Production Spectrum From Galactic Cosmic-Ray Interactions for Gamma-Ray Energies Less Than 1 GeV

5-1 INTRODUCTION

In chapter 1, general formulas were derived for the calculation of the γ-ray spectrum due to secondary particles produced in cosmic-ray collisions with atoms of the interstellar or intergalactic gas.

This chapter will deal with a calculation of the γ-ray spectrum for $E_\gamma < 1$ GeV. In this energy region the analysis is complicated by the lack of a complete understanding of the various modes of pion production. Therefore, the analysis will be carried out in two steps. The first step will be a discussion of the general properties of γ-ray spectra from neutral pion decay, with a determination of the various features of such spectra that must, in general, hold true. This discussion will include the results of analytic and numerical calculations of normalized γ-ray spectra from ideal neutral pion energy distributions. Such results serve two purposes:

(1) They add to an understanding of the salient features of γ-ray spectra from neutral pion decay.
(2) They serve as a test for the accuracy of similar numerical computations where an analytic comparison cannot be obtained.

The general features of γ-ray spectra resulting from neutral pion decay should be kept in mind whenever a discussion of a particular result is desired. The general results will prove invaluable as a guide in discussing the results of the particular model used in the numerical calculations.

Following the general analysis of γ-ray spectra from neutral pion decay, a specific model will be chosen that will be used for numerical calculation of the γ-ray spectrum below 1 GeV. This model is of necessity an oversimplification of the true physics of pion production, but it can be made to fit the present accelerator data to an extent that is adequate for our purposes. The model is of the "isobar-plus-fireball" type. The terms "isobar" and "fireball" are here loosely defined to characterize the main features of the model. The model assumes that at accelerator energies

there exist two dominant modes of pion production. Thus we may speak of two pion components: The isobar component, which dominates pion production in collisions where the cosmic-ray protons have energies in the low GeV range; and the fireball component, which supplies the main bulk of pions for $p-p$ collisions of initial energy greater than 5 GeV.

5-2 GENERAL PROPERTIES OF THE GAMMA-RAY SPECTRUM FROM THE DECAY $\pi^0 \rightarrow \gamma + \gamma$

More neutral pions are produced in high-energy collisions than any other γ-ray-producing secondary particle. We will therefore turn our attention to the properties of the γ-ray spectrum produced by the decay

$$\pi^0 \rightarrow \gamma + \gamma \tag{5-1}$$

We will consider a normalized distribution of neutral pions; i.e., $f(E_\pi)$ such that

$$\int dE_\pi f(E_\pi) = 1 \tag{5-2}$$

We have already shown in chapter 1 how the γ-ray spectrum is related to the neutral pion spectrum. In terms of normalized distribution functions, this relation may be expressed as

$$F(E_\gamma) = \int_\epsilon^\infty dE_\pi \frac{f(E_\pi)}{(E_\pi^2 - m_\pi^2)^{1/2}} \tag{5-3}$$

where $F(E_\gamma)$ is a normalized γ-ray distribution function resulting from the decay of neutral pions, and

$$\epsilon = E_\gamma + \frac{m_\pi^2}{4E_\gamma} \tag{5-4}$$

(See derivation of eq. (1-100).)

Equation (5-4) can be written in the quadratic form

$$E_\gamma^2 - \epsilon E_\gamma + \frac{m_\pi^2}{4} = 0 \tag{5-5}$$

This equation has two roots, $E_{\gamma 1}$ and $E_{\gamma 2}$, such that

$$\epsilon(E_{\gamma 1}) = \epsilon(E_{\gamma 2}) \tag{5-6}$$

Since these roots are solutions of a quadratic equation given by equation (5–5), they must satisfy the condition

$$E_{\gamma 1} \cdot E_{\gamma 2} = \frac{m_\pi^2}{4} \tag{5–7}$$

We will find it useful to express equation (5–7) in logarithmic form and to define

$$\nu \equiv \ln \frac{m_\pi}{2}$$

$$\delta_1 \equiv \nu - \ln E_{\gamma 1}$$

and

$$\delta_2 \equiv \nu - \ln E_{\gamma 2} \tag{5–8}$$

Equation (5–7) then reduces to

$$\delta_1 = -\delta_2 \tag{5–9}$$

or

$$|\delta_1| = |\delta_2| \equiv \delta \tag{5–10}$$

It follows from equation (5–6) that

$$\epsilon(\nu - \delta) = \epsilon(\nu + \delta) \tag{5–11}$$

and therefore

$$F(\nu - \delta) = F(\nu + \delta) \tag{5–12}$$

where the function F is now expressed in terms of $\ln E_\gamma$. Equation (5–12) expresses a useful symmetry on a graph of $F(\ln E_\gamma)$. On a graph of this type, the curve defined by $F(\ln E_\gamma)$ is symmetric about the line $\ln E_\gamma = \nu$ and thus reflects into itself about this line. This property is illustrated in figure 5–1.

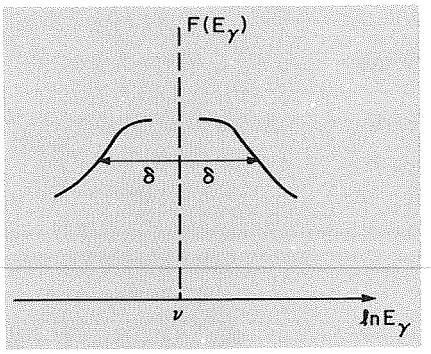

FIGURE 5–1.—A schematic representation of the form of $F(\ln E_\gamma)$ expressing the implications of equation (5–12).

Equation (5–11) can also be expressed as

$$F(E_\gamma) = F\left(\frac{m_\pi^2}{4E_\gamma}\right) \tag{5–13}$$

which is easily seen to be a direct consequence of equations (5–4) and (5–5). Thus, for the important asymptotic cases where the γ-ray spectral distribution function can be approximated by a power law with exponent Γ of the form

$$F(E_\gamma) \rightarrow KE_\gamma^\Gamma \qquad \text{for } E_\gamma > \tilde{E} \tag{5–14}$$

which we will later show to be of astrophysical significance, it follows that

$$F(E_\gamma) \rightarrow K'E^\Gamma \qquad \text{for } E_\gamma < \frac{m_\pi^2}{4\tilde{E}} \tag{5–15}$$

where

$$K' = K(\tfrac{1}{2}m_\pi)^{-2\Gamma} \tag{5–16}$$

Let us now define

$$G(E_\pi) = \frac{f(E_\pi)}{(E_\pi^2 - m_\pi^2)^{1/2}} \tag{5–17}$$

From physical restrictions, it follows that

$$f(E_\pi) \geq 0 \tag{5–18}$$

and

$$(E_\pi^2 - m_\pi^2)^{1/2} \geq 0 \tag{5–19}$$

so that

$$G(E_\pi) \geq 0 \tag{5–20}$$

We can now rewrite equation (5–3) as the functional relation

$$F[\epsilon] = \int_\epsilon^\infty dt\ G(t) \tag{5–21}$$

where t is a dummy variable of integration.

It follows that

$$F[\epsilon_\alpha] = F[\epsilon_\beta] + \int_{\epsilon_\alpha}^{\epsilon_\beta} dt\ G(t) \tag{5–22}$$

where

$$\epsilon_\alpha < \epsilon_\beta$$

and, from equation (5–20),

$$F[\epsilon_\alpha] \geq F[\epsilon_\beta] \qquad \text{for } \epsilon_\alpha < \epsilon_\beta \tag{5–23}$$

Equation (5–23) shows that $F[\epsilon]$ is a monotonically decreasing functional of ϵ.

In order to relate $F[\epsilon]$ to $F(E_\gamma)$, we must consider the quantity

$$\epsilon' \equiv \frac{\partial \epsilon}{\partial E_\gamma} \tag{5–24}$$

From equation (5–4), it follows that

$$\epsilon' = 1 - \frac{m_\pi^2}{4E_\gamma^2} \tag{5–25}$$

Using equations (5–8) and (5–25), we obtain

$$\epsilon' < 0 \qquad \text{for } \ln E_\gamma < \nu$$

$$\epsilon' = 0 \qquad \text{for } \ln E_\gamma = \nu$$

and

$$\epsilon' > 0 \qquad \text{for } \ln E_\gamma > \nu \tag{5–26}$$

We now write

$$\frac{\partial \epsilon}{\partial \delta} = \epsilon' \frac{\partial E_\gamma}{\partial \delta} \tag{5–27}$$

so that

$$\epsilon_\beta = \epsilon_\alpha + \Delta\epsilon = \epsilon_\alpha + \frac{\partial \epsilon}{\partial \delta} \Delta\delta \tag{5–28}$$

From equations (5–8) and (5–10), it follows that

$$\frac{\partial E_\gamma}{\partial \delta} > 0 \qquad \text{for } \ln E_\gamma = \nu + \delta$$

and (5–29)

$$\frac{\partial E_\gamma}{\partial \delta} < 0 \qquad \text{for } \ln E_\gamma = \nu - \delta$$

Thus, from equations (5–26), (5–28), and (5–29), we obtain

$$\frac{\partial \epsilon}{\partial \delta} = 0 \qquad \text{for } \ln E_\gamma = \nu \tag{5–30}$$

and

$$\frac{\partial \epsilon}{\partial \delta} > 0 \qquad \text{at all other points} \tag{5–31}$$

This result, combined with equations (5–26) and (5–27), yields the conclusion that ϵ increases monotonically with δ. Since $F[\epsilon]$ is a monotonically decreasing functional of ϵ, it follows that $F[\delta]$ is a monotonically decreasing functional of δ. Thus, $F[\delta]$ is a maximum at $\delta=0$ and decreases more and more with increasing δ. It follows that $F(E_\gamma)$ is a maximum at $\ln E_\gamma=\nu$; i.e., at $E_\gamma=\frac{1}{2}m_\pi$ and that this is, in fact, the only maximum. Note that these results were reached by less rigorous arguments in our general discussion in section 1–6.

Our results are illustrated in figure 5–2, which shows the general form of $F(E_\gamma)$.

We will further illustrate our results with two ideal distribution functions of pion energy.

Distribution A:

$$f(E_\pi)=\delta(E_\pi-E_0)$$

In this case,

$$F(E_\gamma)=\int_\epsilon^\infty dE_\pi \frac{\delta(E_\pi-E_0)}{(E_\pi^2-m_\pi^2)^{1/2}}$$

$$=\begin{cases}\dfrac{1}{(E_0^2-m_\pi^2)^{1/2}} & \text{for } E_0>\epsilon \\ 0 & \text{otherwise}\end{cases} \qquad (5\text{–}32)$$

To find the limits in terms of E_γ, we need only solve equation (5–5) to obtain

$$E_0>\epsilon \quad \text{implies } E_{\gamma 1}\leq E_\gamma \leq E_{\gamma 2}$$

where

$$E_{\gamma 1}=\tfrac{1}{2}[E_0-(E_0^2-m_\pi^2)^{1/2}]$$

and

$$E_{\gamma 2}=\tfrac{1}{2}[E_0+(E_0^2-m_\pi^2)^{1/2}] \qquad (5\text{–}33)$$

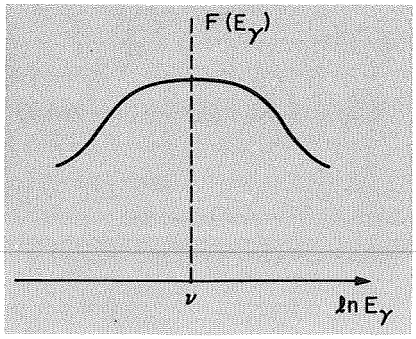

FIGURE 5–2.—A schematic representation of the form $F(\ln E_\gamma)$ expressing the implications of equations (5–12), (5–23), and (5–31).

Distribution B:

$$f(E_\pi) = \begin{cases} (E_B - E_A)^{-1} & \text{for } E_A \leq E_\pi \leq E_B \\ 0 & \text{otherwise} \end{cases}$$

In this case,

$$F(E_\gamma) = \begin{cases} (E_B - E_A)^{-1} \int_{E_A}^{E_B} \dfrac{dE_\pi}{(E_\pi^2 - m_\pi^2)^{1/2}} & \text{for } E_A \geq \epsilon \\ (E_B - E_A)^{-1} \int_{\epsilon}^{E_B} \dfrac{dE_\pi}{(E_\pi^2 - m_\pi^2)^{1/2}} & \text{for } E_A \leq \epsilon \leq E_B \\ 0 & \text{for } \epsilon \geq E_B \end{cases} \tag{5-34}$$

The integral

$$\int \frac{dt}{(t^2 - m^2)^{1/2}} = \ln\,[t + (t^2 - m^2)^{1/2}] + \text{constant} \tag{5-35}$$

Therefore, if we define

$$\begin{aligned} E_{A1} &= \tfrac{1}{2}[E_A - (E_A^2 - m_\pi^2)^{1/2}] & E_{A2} &= \tfrac{1}{2}[E_A + (E_A^2 - m_\pi^2)^{1/2}] \\ \text{and} & & & \qquad (5\text{-}36) \\ E_{B1} &= \tfrac{1}{2}[E_B - (E_B^2 - m_\pi^2)^{1/2}] & E_{B2} &= \tfrac{1}{2}[E_B + (E_B^2 - m_\pi^2)^{1/2}] \end{aligned}$$

we find

$$F(E_\gamma) = \begin{cases} (E_B - E_A)^{-1} \ln\left(\dfrac{E_{B2}}{E_{A2}}\right) & \text{for } E_{A1} \leq E_\gamma \leq E_{A2} \\ (E_B - E_A)^{-1} \ln\left(\dfrac{E_{B2}}{E_\gamma}\right) & \text{for } E_{B1} \leq E_\gamma \leq E_{A1} \text{ and } E_{A2} \leq E_\gamma \leq E_{B2} \\ 0 & \text{otherwise} \end{cases} \tag{5-37}$$

These results are shown schematically in figure 5–3.

5-3 THE ISOBAR-PLUS-FIREBALL MODEL

In this model, we will assume that in collisions at accelerator energies all pions are produced in either of two ways:

(1) Via intermediate production and decay of the Δ(1.238) nonstrange isobar (sometimes also referred to as an excited baryon or baryon resonance). The isobar is assumed to carry momentum directly forward or backward in the cms of the collision with an equal

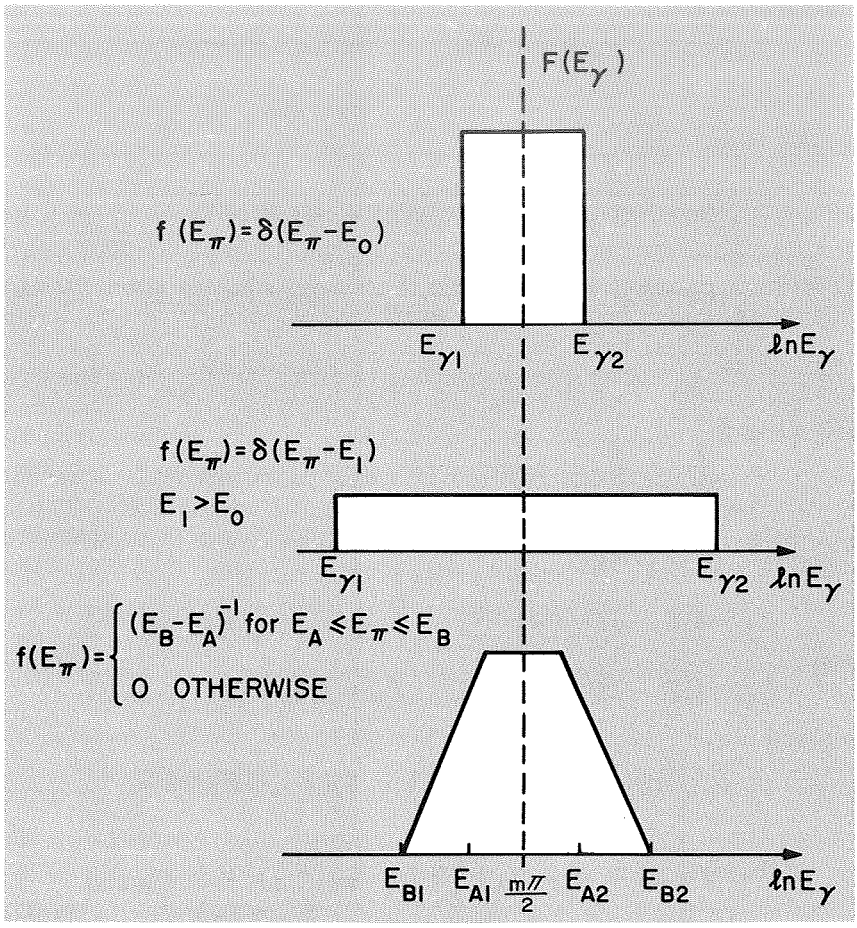

FIGURE 5–3.—Some ideal γ-ray spectra resulting from the decay of some ideal spectra of neutral pions.

probability. This property of isobar production in nucleon-nucleon interactions is well indicated from accelerator studies and has been discussed theoretically (see, e.g., the review by D. R. O. Morrison, 1963).

(2) From the energy remaining available in the cms of the collision, a thermal pion "gas" is created where the pions are given an energy distribution which is very similar to a Maxwell-Boltzmann distribution (Morrison, 1963). This phenomenon sometimes is referred to as a fireball in cosmic-ray nuclear emulsion studies and will be referred to as a fireball here. Its conception dates back to some of Fermi's early research into the problem of pion production in cosmic-ray collisions (Fermi, 1951). Our low-energy fireball is

assumed to be roughly at rest with respect to the cms of the collision with its pions directed isotropically in this system. As an illustration, figure 5–4 shows schematically the situation resulting from a TeV (10^3 GeV) collision of the type we will discuss in the next chapter. The large difference in the behavior of the mesons and baryons emanating from the collision leads to asymptotic forms for the high-energy γ-ray spectrum described in that chapter.

The situation for low-energy collisions is not that simple, as is shown in figure 5–5. As is indicated, it is much harder to distinguish between the two modes of pion production because of the similarity in the momenta involved. There is no large disparity in cms momenta as in the case of a TeV collision.

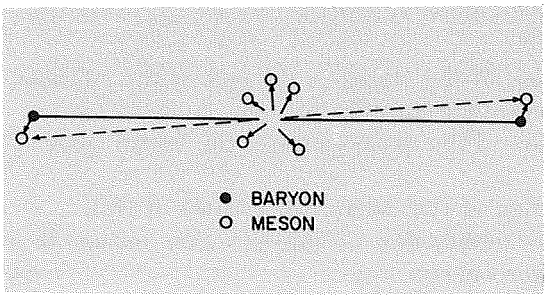

FIGURE 5–4.—A schematic representation of the distribution of secondary pions resulting from a p–p collision in the teraelectron volt energy range.

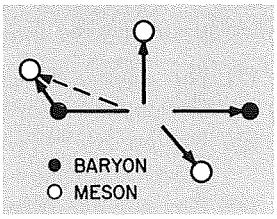

FIGURE 5–5.—A schematic representation of the distribution of secondary pions resulting from a p–p collision in the gigaelectron volt energy range.

Indeed, we may even conceptually fuse the two components and envision all the created pions together as having an elongated cms momentum distribution whose anisotropy is a function of collision energy. A simple model of this type has been used by Hayakawa et al. (1964) in their calculations of the cosmic γ-ray spectrum. However, the two-component model presented here utilizes the more detailed results of more recent accelerator experiments. Among the most striking of these results is the complete dominance of the $\Delta(1.238)$ production mode in collisions where $E_p < 3$ GeV (Muirhead, 1965 (p. 666)). Indeed, even for collisions below the threshold for production of the mass peak at 1.238 GeV, there is evidence that this resonance (whose mass distribution extends below 1.238 GeV) affects the pion energy distribution (Focardi et al., 1965). We will assume here that below 3.16 GeV, all pion production occurs through the intermediate 1.238 resonance. This hypothesis is in agreement with the observation that at about 0.6-GeV proton kinetic energy, the neutral pion distribution is

isotropic, since in this energy range the isobar enhancement would be formed at rest in the cms. The model is also in agreement with the observation of an increasing asymmetry in pion momentum toward the forward and backward directions as E_p increases, since at higher energies the isobars tend to carry more and more momentum in the forward and backward directions.

5–4 CALCULATION OF THE GAMMA-RAY SPECTRUM FROM THE ISOBAR PION COMPONENT

We are now ready to specify the parameters needed for the numerical calculation of the γ-ray spectrum from the isobar pion component, hereafter referred to as the i-process component. For our model we need to specify the following:

(1) The energy distribution of the isobar in the cms of the collision.
(2) The angular distribution of the isobar in the cms.
(3) The energy distribution of the decay pions in the BARS (baryon-isobar rest system).
(4) The angular distribution of the decay pions in the BARS.
(5) The cross section for production of the $\Delta(1.238)$ isobar as a function of initial proton energy.

In our model the isobars are assumed to decay isotropically in the BARS. The energy distribution of the decay pions is obtained from the Breit-Wigner form of the isobar mass distribution. The isobar mass distribution is assumed to peak at $M_0^*=1.238$ GeV and have a width of 0.1 GeV. The Breit-Wigner distribution is given by

$$\mathscr{F}(M^*)=\frac{1}{\pi}\frac{\Gamma}{(M^*-M_0^*)^2+\Gamma^2} \tag{5–38}$$

We denote the pion energy in the BARS by the symbol μ, which was used in chapter 1. In terms of isobar mass, μ is then given by

$$\mu=\frac{M^{*2}+M_\pi^2-M_p^2}{2M^*} \tag{5–39}$$

The normalized distribution function for μ is related to the normalized Breit-Wigner mass distribution by the transformation

$$\mathscr{F}(\mu)=\frac{\mathscr{F}(M^*)}{d\mu/dM^*} \tag{5–40}$$

which yields

$$\mathscr{F}(\mu)=\frac{\Gamma}{\pi}\left\{[\{\mu+[\mu^2+(M_p^2-M_\pi)^2]^{1/2}-M_0^*\}^2+\Gamma^2]\left[1-\frac{\mu}{\mu+(\mu^2+M_p^2-M_\pi^2)^{1/2}}\right]\right\}^{-1} \tag{5-41}$$

Equation (5–41) was solved numerically by the author (Stecker, 1970) as a check on an intermediate step in the complete numerical solution. The results are graphed in figure 5–6 along with the BARS γ-ray distribution, which results from this pion distribution and is given by

$$f(E'_\gamma)=\int_{E'_\gamma+(M_\pi^2/4E'_\gamma)}^{\infty}\frac{d\mu\,\mathscr{F}(\mu)}{(\mu^2-M_\pi^2)^{1/2}} \tag{5-42}$$

In the actual calculation of the γ-ray spectrum, we must take into account the kinematic limits on the allowed pion energy. There is only a limited amount of energy available for pion production. In order to specify a maximum BARS pion energy $\mu_{\max}$, we note that in the extreme case, where all the cms energy of the collision is transformed into the mass of the resonance,

$$M^*_{\max}=(2\gamma_c-1)\,M_p \tag{5-43}$$

Thus we can define

$$\mu_{\max}=\frac{M^{*2}_{\max}+M_\pi^2-M_p^2}{2M^*_{\max}} \tag{5-44}$$

We must then renormalize the distribution function over μ so that its integral over the allowed energy region is equal to unity. We thus define a renormalizing weighting factor

$$w_{\rm rn}\equiv\left[\int_{m_\pi}^{\mu_{\max}(E_p)}d\mu\;\mathscr{F}(\mu)\right]^{-1} \tag{5-45}$$

which multiplies the distribution $\mathscr{F}(\mu)$.

We specify in our model, as before, that the isobars carry momentum either directly forward or directly backward in the cms (Morrison, 1963). We further assume that the low-energy *i*-process produces pions through the two-stage decay

$$\begin{array}{l} p+p\rightarrow p+\Delta^+(1.238) \\ \qquad\qquad\quad \hookrightarrow p+\pi^0 \\ \qquad\qquad\qquad\quad \hookrightarrow 2\gamma \end{array} \tag{5-46}$$

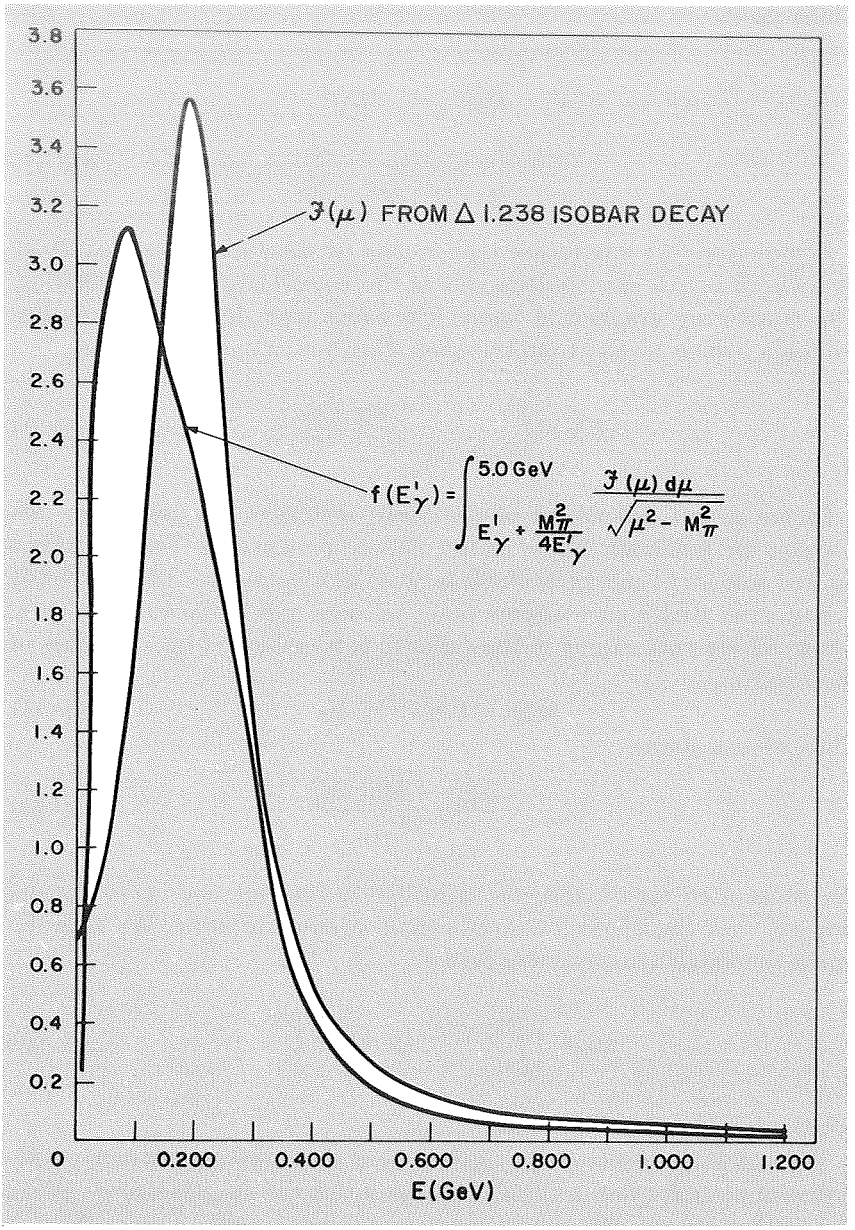

FIGURE 5–6.—The energy distribution function of pions in the BARS resulting from the decay of the Δ(1.238) isobar and the energy distribution function of γ-rays in the BARS resulting from the decay of these pions (Stecker, 1970).

Let E_c be the total energy of the collision in the cms; i.e.,

$$E_c = 2M_p\gamma_c \tag{5–47}$$

Furthermore, let us designate the energy of the isobar and proton in the cms system of the collision by E_{ci} and E_{cp}, respectively. It has been shown in chapter 1 (see eq. (1–62)) that

$$E_{ci}=\frac{E_c^2+M^{*2}-M_p^2}{2E_c} \tag{5–48}$$

The energy of the forwardly and backwardly produced isobars in the laboratory system are then

$$E_f=\gamma_c(E_{ci}+\beta_c p_{ci}) \tag{5–49}$$

and

$$E_b=\gamma_c(E_{ci}-\beta_c p_{ci}) \tag{5–50}$$

with

$$p_{ci}=(E_{ci}^2-M^{*2})^{1/2} \tag{5–51}$$

We may therefore write

$$\sigma(E_i|E_p)=\frac{\sigma_i(E_p)}{2}\left[\delta(E_i-E_f)+\delta(E_i-E_b)\right] \tag{5–52}$$

We now wish to determine the normalized γ-ray energy distribution function for the i-process decay of the $\Delta(1.238)$ isobar. As we showed in our discussion of equation (1–14), since the i-process is a two-stage process, the determination of the distribution function $f_{di}(E_\gamma|E_i)$ requires an integration over an additional energy parameter. We may, if we wish, specify this parameter as E_π as in chapter 1; however, since type i decays are symmetric in the BARS, it will be simpler to use E'_γ (the energy of the decay γ-ray in the BARS). We can then specify

$$f_{di}(E_\gamma|E_i)=\int dE'_\gamma f(E'_\gamma) f_i(E_\gamma|E_i, E'_\gamma) \tag{5–53}$$

In the first stage of the decay, the energy of the resultant neutral pion in the BARS is again uniquely determined and is given by

$$\mu_i=\frac{M_i^{*2}+M_\pi^2-M_N^2}{2M^*} \tag{5–54}$$

Therefore, the magnitude of the BARS momentum of the neutral pion is also unique. We can thus define the convenient parameter ρ_i by the relation

$$\rho_i\equiv\frac{p_{\pi,i}}{M^*}=\frac{(\mu_i^2-M_\pi^2)^{1/2}}{M^*} \tag{5–55}$$

In the pion rest system, each of the two decay γ-rays is given an energy $\frac{1}{2}M_\pi$. Thus, in transforming to the BARS, we find

$$E'_\gamma = \gamma'_\pi \tfrac{1}{2} M_\pi (1 + \beta'_\pi \cos\phi) = \tfrac{1}{2}(\mu_i + \rho_i M_i^* \cos\phi) \tag{5-56}$$

where ϕ is the angle between the γ-ray momentum vector in the pion rest system and the pion momentum vector in the BARS.

Once again, the decay is isotropic in the rest system of the decaying particle (i.e., in cos ϕ), so we obtain

$$f(E'_\gamma) = \begin{cases} \dfrac{1}{\rho_i M_i^*} & \text{for } L_i \leq E'_\gamma \leq U_i \\ 0 & \text{otherwise} \end{cases} \tag{5-57}$$

where, from equation (5–56), we find

$$L_i \equiv \tfrac{1}{2}(\mu_i - \rho_i M_i^*)$$

and (5–58)

$$U_i \equiv \tfrac{1}{2}(\mu_i + \rho_i M^*)$$

Once E'_γ is specified, we need only extend the reasoning used in the derivation of equation (1–106) to obtain

$$f(E_\gamma | E_i, E'_\gamma) = \begin{cases} \dfrac{M^*}{2E'_\gamma (E_i^2 - M^{*2})^{1/2}} & \text{for } E_{\gamma 1} \leq E_\gamma \leq E_{\gamma 2} \\ 0 & \text{otherwise} \end{cases} \tag{5-59}$$

so that as $\beta_i \to 1$,

$$E_{i,\,\max}(E_\gamma) \to \infty$$

and (5–60)

$$E_{i,\,\min}(E_\gamma) \to \left(\frac{M}{2E'_\gamma}\right) E_\gamma$$

Through the use of equation (1–8), equation (1–10) in this case reduces to

$$I_i(E_\gamma) = \tfrac{1}{2}\langle nL \rangle \int dE_p I(E_p) w_{\rm rn}(E_p) \int dE_i \left[\frac{M^*}{(E_i^2 - M^{*2})^{1/2}}\right] \sigma(E_i | E_p) R_i \times \int_{L(E_\gamma, E_i)}^{U(E_\gamma, E_i)} \frac{dE'_\gamma}{E'_\gamma} \int_{E'_\gamma + (M_\pi^2/4E'_\gamma)}^{\mu_{\max}(E_p)} d\mu \frac{\mathscr{F}(\mu)}{(\mu^2 - M_\pi^2)^{1/2}} \tag{5-61}$$

where $w_{\rm rn}$ is defined by equation (5–45), $\mu_{\max}$ is defined by equation (5–44), E'_γ is the γ-ray energy in the BARS, $R_i = 2/3$ is the branching

ratio for the neutral pion decay mode of the $\Delta(1.238)$ isobar,

$$U(E_\gamma, E_i) = \frac{E_\gamma}{\gamma_i(1-\beta_i)} \tag{5-62}$$

$$L(E_\gamma, E_i) = \frac{E_\gamma}{\gamma_i(1+\beta_i)} \tag{5-63}$$

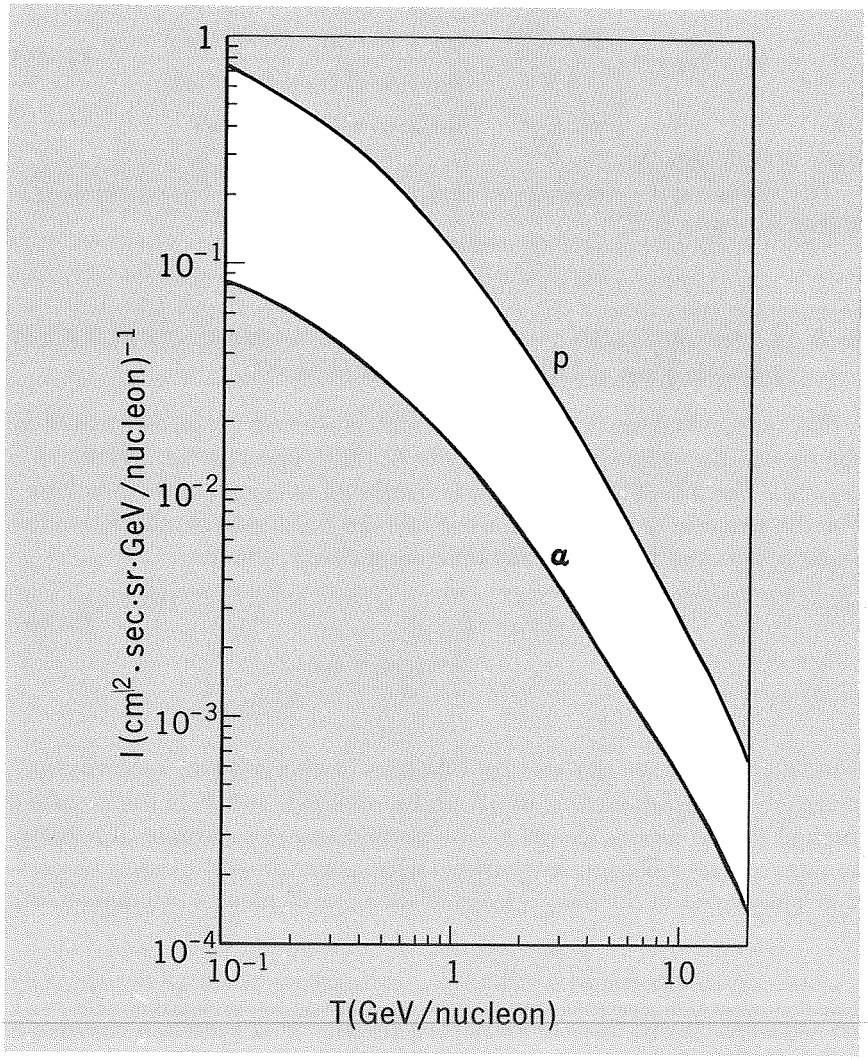

FIGURE 5–7.—The differential cosmic-ray proton and alpha particle kinetic-energy spectra.

and

$$\langle nL \rangle \equiv \int_0^{L_{\text{eff}}} dr\, n(\mathbf{r}) \tag{5-64}$$

with the cosmic-ray intensity in the galaxy $I(E_p)$ assumed to be constant.

The integrations in equation (5–61) were carried out numerically by the author (Stecker, 1970). The product $R_i\sigma_i(E_p)$ (in millibarns) was approximated by a function which was taken to be a three-part power law of the form

$$R_i\sigma_i(E_p) = \begin{cases} 0 & \text{for} \quad E_p < 1.48 \text{ GeV} \\ 10^{-1}\, E_p^{6.7} & \text{for } 1.48 < E_p < 1.88 \text{ GeV} \\ 6.9 & \text{for } 1.88 < E_p < 3.16 \text{ GeV} \\ 69\, E_p^{-2} & \text{for } 3.16 < E_p < 10 \text{ GeV} \end{cases} \tag{5-65}$$

with σ_i in millibarns.[1]

The differential cosmic-ray proton intensity used in the calculation is shown in figure 5–7.

5–5 CALCULATION OF THE GAMMA-RAY SPECTRUM FROM THE FIREBALL COMPONENT

We now calculate the γ-ray spectrum produced by pions created in the cms of the collision. We will refer to these pions as the fireball component. (The use of the term here is restricted to our previous definition.) For the purpose of discussion, we define the f-process (as opposed to the i-process) as the following production and decay scheme:

$$\begin{aligned} p + p &\rightarrow p + p + F \\ &\hookrightarrow \zeta_{\pm}\pi^{\pm} + \zeta_0\pi^0 \\ &\hookrightarrow 2\zeta_0\gamma \end{aligned} \tag{5-66}$$

where $\zeta_{\pm}$ and ζ_0 are the average numbers of charged and neutral pions, respectively (a function of initial proton energy), and F is the so-called fireball. By no means should F be regarded as a real particle; it is better to picture it as a bosonic nonparticle whose only properties are a momentary possession of the mass energy of the created pions and a momentary

[1] Such a cross section is an approximation which assumes complete dominance of the i-process for neutral pion production in collisions involving kinetic energies below 1 GeV but which maintains the observed E^{-2} energy dependence of the $\Delta(1.238)$ cross section in various channels for kinetic energies above 2 GeV (Muirhead, 1965; Ellis et al., 1968; and Connolley et al., 1967).

occupation of the interaction volume centered at the center of momentum of the collision and therefore moving at β_c with respect to the ls.

We assume that the fireball decays isotropically with a cms energy distribution which is roughly independent of energy. The justification for this assumption comes both from accelerator studies (Morrison, 1963, and Fidecaro et al., 1962) and from studies of interactions of cosmic rays in the atmosphere (Fowler and Perkins, 1964). Accelerator studies (Morrison, 1963) of p–p interactions for energies up to 30 GeV give a charged-pion transverse-momentum distribution that fits an empirical law of the form

$$f_\pi(p_\perp) \propto p_\perp e^{-2p_\perp/\langle p_\perp\rangle_\pi} \tag{5–67}$$

Studies at 300 GeV (Guseva et al., 1962) fit this law very well. (There is only a change in normalization involved, indicating an increased multiplicity.) It has also been observed that $\langle p_\perp\rangle_\pi$ is roughly constant in p–p collisions involving energies of up to 10^4 GeV (Fowler and Perkins, 1964). If neutral pions also obey this law, we would expect that the resultant γ-rays would have a transverse-momentum distribution of the form

$$f_\gamma(p_\perp) \propto e^{-p_\perp/\langle p_\perp\rangle_\gamma} \tag{5–68}$$

with

$$\langle p_\perp\rangle_\gamma = \tfrac{1}{2}\langle p_\perp\rangle_\pi \tag{5–69}$$

Data confirm this exponential distribution for $p_\perp > 0.1$ GeV in p–p collisions involving energies up to 10^4 GeV (Fowler and Perkins, 1964). Accelerator experiments (Fidecaro et al., 1962) measuring the γ-ray spectrum from p–p interactions at 23.1 GeV have substantiated indications of an exponential distribution of neutral pions in the cms. The data are well represented by an energy distribution function of the form

$$f(E_\gamma^{(c)}) \propto e^{-4E_\gamma^{(c)}} \tag{5–70}$$

Figure 5–8 shows the average cms energy distribution of γ-rays produced in 23.1 GeV p–p interactions as measured by Fidecaro et al. (1962).

Thus, over a wide range of energies

$$f_\pi(p_\perp) \propto p_\perp e^{-2p_\perp/\langle p_\perp\rangle_\pi} \tag{5–71}$$

The quantity $p_\perp$ is a relativistic invariant that is proportional to the cms momentum of the pion and, therefore (relativistically), to its cms

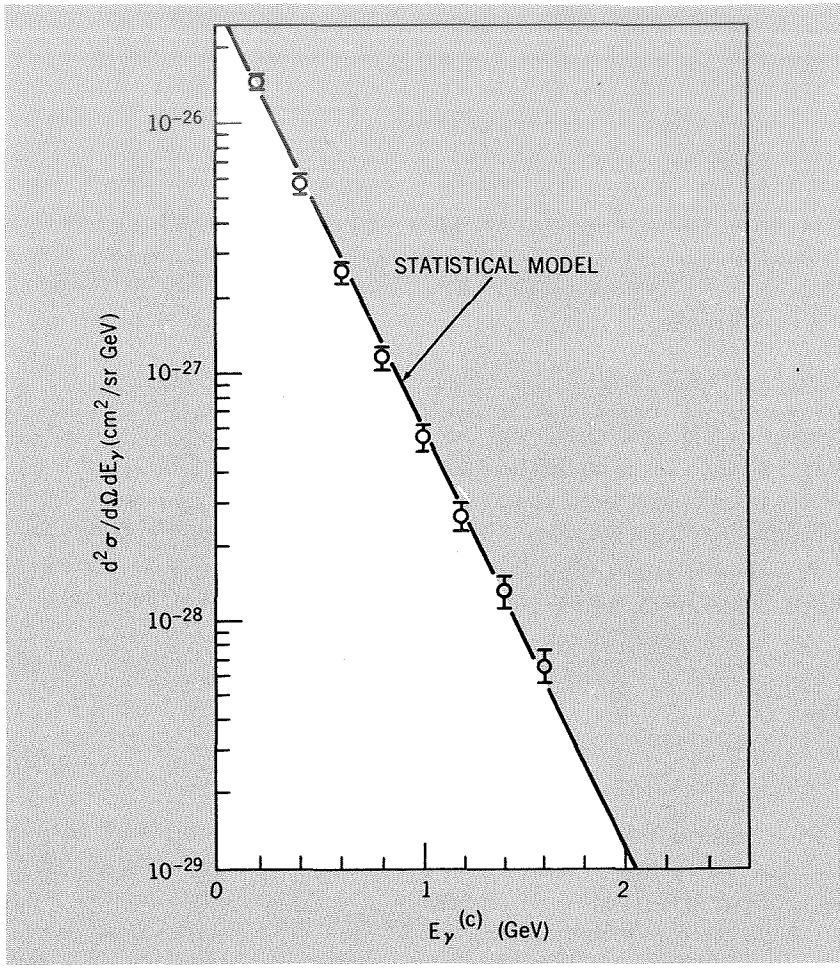

FIGURE 5–8.—Gamma-ray cms energy spectrum integrated over angles as given by Fidecaro et al. (1962). The solid line has a slope of −3.86.

energy. Letting ξ stand for cms pion energy, we may therefore use the approximation

$$\mathscr{F}(\xi) \propto \xi e^{-2\xi/\langle\xi\rangle} \tag{5–72}$$

Since $\langle\xi\rangle \simeq 0.5$, independent of energy for collisions involving energies up to 10^4 GeV, we find that

$$\mathscr{F}(\xi) = \frac{1}{Z}\,\xi e^{-4\xi} \tag{5–73}$$

with the normalization constant Z defined by

$$Z = \int_{m_\pi}^{\mu_{\max}(E_p)} d\xi \; \xi e^{-4\xi} \tag{5–74}$$

so that

$$\int_{m_\pi}^{\mu_{\max}(E_p)} d\xi \; \mathscr{F}(\xi) = 1 \tag{5–75}$$

Evidence for a two-component pion production model with the f-process component producing pions isotropically in the collision cms has been presented by Dekkers et al. (1965). Their work indicates that the i-component yields a very large positive-to-negative pion ratio. Thus, the negative pions arise principally through the f-process. It is also found that at accelerator energies, most negative pions are produced isotropically in the cms (the remaining ones are consistent with a small contribution from $N^* \rightarrow N\pi\pi$ decays).

We have thus assumed that the i-process plus the f-process provide all the pions produced; i.e.,

$$\zeta_f \sigma_f \equiv [\zeta(E_p)\sigma(E_p)]_{\text{Total}} - \sigma_i(E_p) \tag{5–76}$$

The function $[\zeta(E_p)\sigma(E_p)]_{\text{Total}}$ is shown in figure 5–9.

It has been shown that a γ-ray of energy $E_\gamma^{(c)}$ in the cms may have any laboratory energy E_γ within the range

$$\gamma_c E_\gamma^{(c)}(1 - \beta_c) \leq E_\gamma \leq \gamma_c E_\gamma^{(c)}(1 + \beta_c) \tag{5–77}$$

Under our assumption of uniform cms pion emission for the f-process, there is a constant production probability

$$f(E_\gamma | E_\gamma^{(c)}) = \frac{1}{2\gamma_c \beta_c E_\gamma^{(c)}} \tag{5–78}$$

within this energy range. It follows from equation (5–77) that the integral over the source function with respect to $E_\gamma^{(c)}$ is bounded by the upper and lower limits, U_F and L_F, respectively, where

$$U_F(E_\gamma, E_p) = \frac{E_\gamma}{\gamma_c(1 - \beta_c)} \tag{5–79}$$

and

$$L_F(E_\gamma, E_p) = \frac{E_\gamma}{\gamma_c(1 + \beta_c)} \tag{5–80}$$

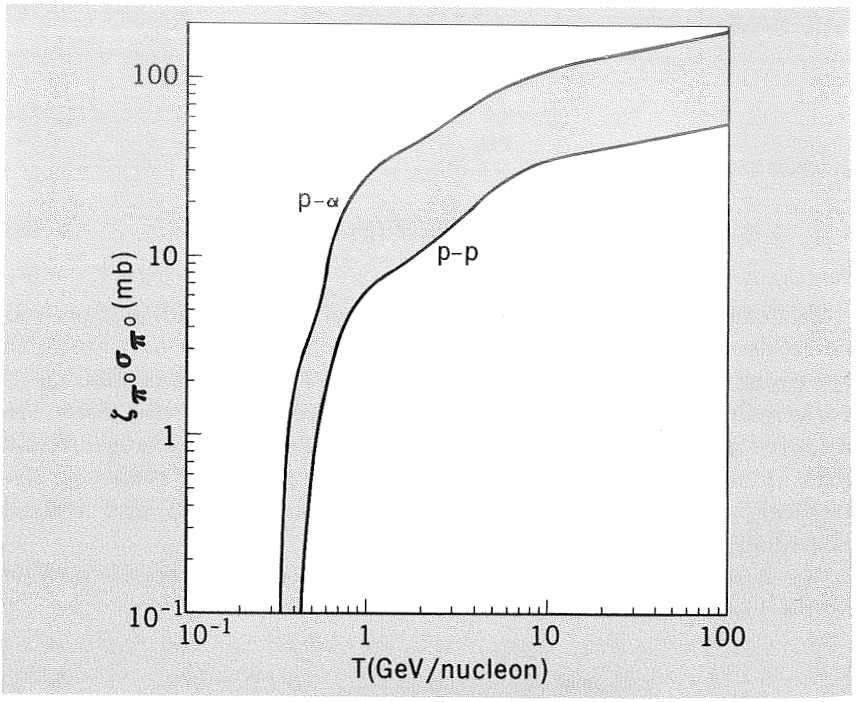

FIGURE 5–9.—Cross section times multiplicity for neutral pion production. Data for p–p interactions as summarized by Muirhead (1965). Data for p–α interactions from Prokoshkin and Tiapkin (1957a, b), Batson et al. (1959), and Kozodaev et al. (1960).

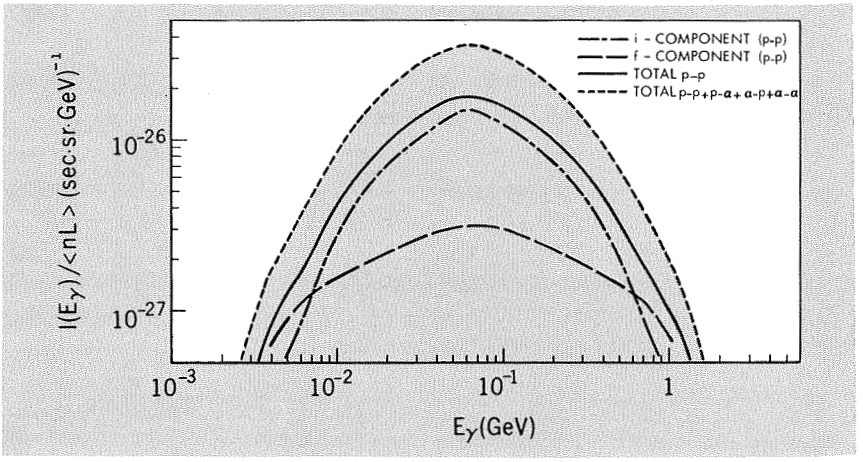

FIGURE 5–10.—The calculated differential production spectrum of γ-rays produced in cosmic-ray interactions based on the "isobar-plus-fireball" model described in the text (Stecker, 1970)

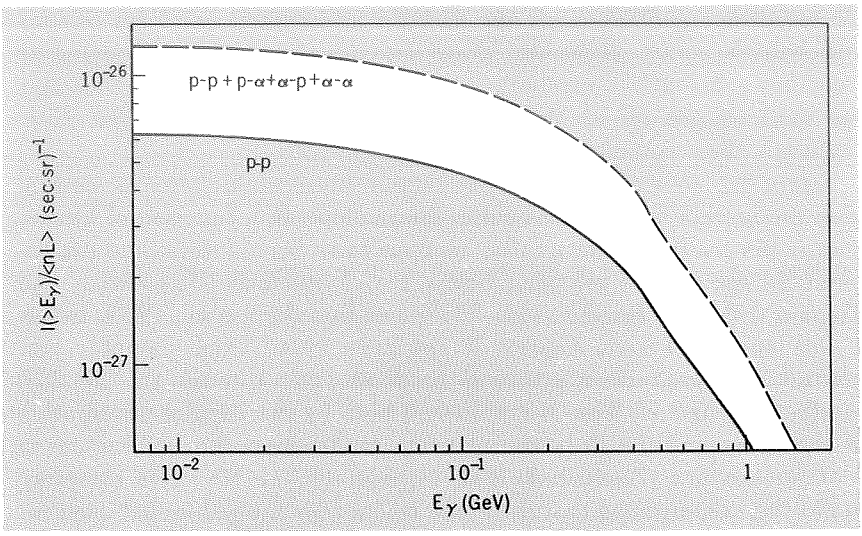

FIGURE 5–11.—Calculated integral production spectrum based on the results of figure 5–6 (Stecker, 1970).

since only γ-rays within this cms energy range will produce γ-rays of laboratory energy E_γ.

Therefore, the complete formula for the γ-ray spectrum arising from the f-process is

$$I_f(E_\gamma) = \frac{\langle nL\rangle}{2}\int dE_p \frac{\zeta_f(E_p)\sigma_f(E_p)I(E_p)}{\gamma_c(E_p)\beta_c(E_p)} \times \int_{L_r(E_\gamma,E_p)}^{U_F(E_\gamma,E_p)} \frac{dE_\gamma^{(c)}}{E_\gamma^{(c)}} \int_{E_\gamma^{(c)}+(m_\pi^2/4E_\gamma^{(c)})}^{\mu\max(E_p)} d\xi \frac{\mathscr{F}(\xi)}{(\xi^2-m_\pi^2)^{1/2}} \qquad (5\text{–}81)$$

The resultant f-process spectrum was obtained numerically by the same methods previously described for the evaluation of the i-process spectrum (Stecker, 1970). The results of the numerical integration are shown in figure 5–10. Figure 5–10 also shows the total spectrum produced by summation of the i- and f-components. The integral spectrum is given in figure 5–11.

5–6 COSMIC GAMMA RAYS PRODUCED IN COSMIC-RAY p–α AND α–p INTERACTIONS

Up to this point, we have restricted ourselves to a discussion of cosmic γ-ray production in cosmic-ray p–p interactions. The details of particle production in p–p interactions have been well studied in recent accelera-

tor experiments. Such is not the case for interactions involving protons and helium nuclei. However, these interactions may be expected to provide a substantial contribution to the total γ-ray production of roughly the same magnitude as the contribution from p–p interactions. Consequently these interactions cannot be neglected.

Figure 5–9 shows the total cross section times multiplicity for neutral pion production and proton-helium interactions as a function of kinetic energy per nucleon. These cross sections have been determined experimentally for kinetic energies below 1 GeV/nucleon (Prokoshkin and Tiapkin, 1957a, b; Batson et al., 1959; and Kozodaev et al., 1960). Below 1 GeV/nucleon kinetic energy, it has been found experimentally that neutral pion production proceeds predominantly through the $\Delta(1.238)$ production channel. This is manifested both by the angular distribution of the pions and the 2:1 ratio of neutral-to-positive pion production in p–n interactions (Prokoshkin and Tiapkin, 1957a). We have therefore made the assumptions that the same isobar-plus-fireball model used for p–p interactions is also valid for p–α and α–p interactions, that the i-process dominates below a total energy of 3.16 GeV/nucleon, and that the energy dependences of the cross sections for the i-process and the f-process are the same for nucleon-nucleon collisions in all of these interactions (charge independence). For interactions involving kinetic energies greater than 1 GeV/nucleon, the cross-section data were extrapolated by assuming that the energy dependence of the pion multiplicity has the same form for α–p and p–α interactions as for p–p interactions. We assume also that the cross section for $\Delta(1.238)$ production decreases with energy as E^{-2}, as is the case for p–p interactions. The assumption is also made that the pions are produced primarily in interactions which simulate free nucleon-nucleon interactions (Batson et al., 1959), but with shadowing corrections as given by Kozodaev et al. (1960).

The energy spectrum of cosmic-ray alpha particles at solar minimum is shown in figure 5–7. These particles produce γ-rays by interacting with the interstellar gas (primarily α–p interactions). To this contribution we must add the contribution of cosmic-ray protons interacting with helium nuclei in the interstellar gas (p–α interactions). Interstellar helium is estimated to make up approximately 36 percent of the nucleons in the interstellar medium so that the ratio of helium nucleons to hydrogen nucleons is 0.57 (Allen, 1963).

The differential energy spectrum of cosmic γ-rays from α–p and p–α interactions was calculated based on the intensities and cross sections given in figures 5–7 and 5–9, respectively, and on the assumptions given above (Stecker, 1970). The results of this calculation, when added to the results of the p–p calculations, yield the total spectrum given in figure 5–10. The total integral production spectrum is shown in figure 5–11.